Wavelet-based adaptive solution of the Nonuniform Multiconductor Transmission Lines

Soluzione adattativa basata sulle ondine per le linee multiconduttore non uniformi

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Electrical interconnects like wires, cables, or traces on printed circuit boards provide the basic link between different parts of any electronic system. For low speed systems these structures have an ideal behavior, and do not introduce distortions on the carried signals. However, as the clock frequencies of digital systems increase, the effects of parasitic electromagnetic couplings become more and more important. Some distortions are introduced due to the mutual interaction between different conductors. Therefore, the simulation of these structures has become an important step for the analysis and design of high-speed electronic systems.

We define here an interconnect as a set of $P+1$ conductors, one of which is the reference for voltages and the return for currents, forming a structure with a small cross-section with respect to the smallest wavelength of the surrounding electromagnetic fields. There is no restriction on the length of the conductors. Many interconnects of practical interest are characterized by cross-sections which are not translation-invariant. Examples can be impedance matching networks or cables in complex structures, like automobiles or airplanes. The Nonuniform Multiconductor Transmission Lines (NMTL) equations [1] constitute an appropriate model for such structures. In their non-dimensional and time-explicit form, these equations can be expressed as

$$\frac{\partial}{\partial t}\mathbf{i}(x,t) = -\mathbf{\Gamma}(x)\frac{\partial}{\partial x}\mathbf{v}(x,t) - \mathbf{\Gamma R}(x)\mathbf{i}(x,t), \quad (1)$$

$$\frac{\partial}{\partial t}\mathbf{v}(x,t) = -\mathbf{S}(x)\frac{\partial}{\partial x}\mathbf{i}(x,t) - \mathbf{SG}(x)\mathbf{v}(x,t), \quad (2)$$

where $\mathbf{v}$ and $\mathbf{i}$ are arrays of length P and represent voltage and current on each conductor as functions of the normalized longitudinal coordinate $x \in [0,1]$ and time t. The $P \times P$ matrices $\mathbf{\Gamma}(x)$, $\mathbf{S}(x)$, $\mathbf{\Gamma R}(x)$, and $\mathbf{SG}(x)$ depend on the cross-sectional geometry of the structure. The first two matrices are symmetric and positive definite $\forall x \in [0,1]$.

The set of equations to be solved is complete once the initial and the boundary conditions are specified. In this work we assume zero initial conditions and use linear and resistive terminations,

$$\mathbf{v}(0,t) = \mathbf{v_S}(t) - \mathbf{r_S}\mathbf{i}(0,t), \qquad \mathbf{v}(1,t) = \mathbf{v_L}(t) + \mathbf{r_L}\mathbf{i}(1,t)\,, \tag{3}$$

where $\mathbf{r_S}$ and $\mathbf{r_L}$ are $P \times P$ nonsingular matrices and $\mathbf{v_S}(t)$, $\mathbf{v_L}(t)$ are arrays with the source voltage generators launching the signals on the interconnect. These travelling signals are often characterized by isolated singularities in the first derivative, being regular elsewhere. For this reason, it is convenient to represent them by using adaptive spatial wavelet expansions.

We will work with the construction of biorthogonal wavelets on the unit interval of [2]. This construction is derived from an underlying biorthogonal multiresolution on $\mathbb{R}$ by introducing some modified scaling functions and wavelets at the borders. The "internal" scaling functions and wavelets, characterized by a support strictly included in $[0,1]$, remain unchanged. This is obviously only possible when the refinement level is larger than a minimum level j_0. It is shown in [2] that the features of the underlying multiresolution on $\mathbb{R}$, like stability and approximation properties, are inherited by the multiresolution on $[0,1]$. In addition, at any refinement level j, there is only one scaling function and only one wavelet with a nonvanishing value at each border.

Starting from the minimum level j_0 up to a maximum level J, we have the usual multilevel decomposition for primal and dual spaces

$$V_J = V_{j_0} \oplus \bigoplus_{j=j_0}^{J-1} W_j, \qquad \widetilde{V}_J = \widetilde{V}_{j_0} \oplus \bigoplus_{j=j_0}^{J-1} \widetilde{W}_j, \tag{4}$$

where the scaling functions and the wavelets spaces are, respectively,

$$V_j = \mathrm{span}\{\varphi_{jk},\, k = 0, \ldots, \mathrm{dim}V_j - 1\}, \quad W_j = \mathrm{span}\{\psi_{jk},\, k = 0, \ldots, 2^j - 1\},$$

and similarly for duals. We use the primal hierarchical bases to expand the solution,

$$\begin{aligned}
\mathbf{v}(x,t) &= \sum_{k=0}^{\mathrm{dim}V_{j_0}-1} \varphi_{j_0,k}(x)\check{\mathbf{v}}_{j_0,k}(t) + \sum_{j=j_0}^{J-1}\sum_{k=0}^{2^j-1} \psi_{j,k}(x)\hat{\mathbf{v}}_{j,k}(t),\\
\mathbf{i}(x,t) &= \sum_{k=0}^{\mathrm{dim}V_{j_0}-1} \varphi_{j_0,k}(x)\check{\mathbf{i}}_{j_0,k}(t) + \sum_{j=j_0}^{J-1}\sum_{k=0}^{2^j-1} \psi_{j,k}(x)\hat{\mathbf{i}}_{j,k}(t),
\end{aligned}$$

and we test the NMTL equations with the dual hierarchical basis functions. Due to the biorthogonality between primal and dual functions, the resulting system is explicit in the time derivatives of the unknowns, and can be formally expressed as

$$\frac{d}{dt}\begin{bmatrix}\mathbf{I}\\ \mathbf{V}\end{bmatrix} = \mathbf{A}\begin{bmatrix}\mathbf{I}\\ \mathbf{V}\end{bmatrix},$$

where the arrays **I** and **V** collect the scaling function and wavelet coefficients of current and voltage. The matrix **A** is obtained through integrals of scaling functions and wavelets. These integrals are computed with the algorithm described in [3].

We turn now to the implementation of the boundary conditions in Eq. (3). We will only treat explicitely the left boundary condition, because the same procedure can be applied without modifications for the right one. We begin to note that if we use the canonical bases of V_J and $\widetilde{V}_J$ as trial and test functions, the inclusion of the boundary conditions is rather simple. This is due to the fact that only one scaling function (with $k = 0$) is nonvanishing at the border. Therefore, Eq. (3) translates into a relation involving only one coefficient,

$$\check{\mathbf{v}}_{J,0} = \mathbf{v}_{\mathbf{S}}(t) - \mathbf{r}_{\mathbf{S}}\check{\mathbf{i}}_{J,0}.$$

Defining now the new coefficients

$$\check{\mathbf{a}}_{J,0} = \check{\mathbf{v}}_{J,0} + \mathbf{r}_{\mathbf{S}}\check{\mathbf{i}}_{J,0}, \qquad \check{\mathbf{b}}_{J,0} = \check{\mathbf{v}}_{J,0} - \mathbf{r}_{\mathbf{S}}\check{\mathbf{i}}_{J,0},$$

we see that the boundary conditions are simply expressed as Dirichlet-type conditions on the new coefficient $\check{\mathbf{a}}_{J,0}$. The implementation of this conditions is trivial.

The above procedure cannot be applied directly to the system obtained with the hierarchical bases, because more than one trial function is nonvanishing at the border. However, there is only one nonvanishing border scaling function or wavelet at a fixed refinement level (labelled with $k = 0$). Let us collect all these functions in a border space V_J^0, of which we give two alternative representations,

$$V_J^0 = \mathrm{span}\{\varphi_{j_0,0}, \psi_{j_0,0}, \ldots, \psi_{J-1,0}\} = \mathrm{span}\{\varphi_{J,0}, \psi_{j_0,0}^{\mathrm{mod}}, \ldots, \psi_{J-1,0}^{\mathrm{mod}}\}. \qquad (5)$$

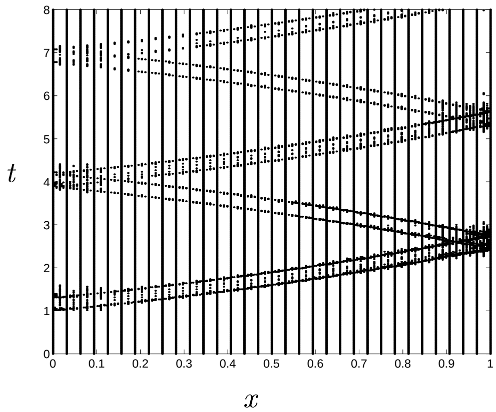
The modified border wavelets are defined as

$$\psi_{j,0}^{\mathrm{mod}} = \psi_{j,0} - \langle \psi_{j,0}, \tilde{\varphi}_{J,0} \rangle \varphi_{J,0}, \qquad j = j_0, \ldots, J-1,$$

they are linearly independent, and vanish at the border. In addition, the basis changes between the two representations are expressed by simple matrices that can be evaluated in a closed form. The only nonvanishing border function in the modified representation is $\varphi_{J,0}$. Therefore, the implementation of the boundary conditions becomes trivial. In addition, the system in the modified hierarchical basis is fully equivalent to the system in the canonical basis, and can be expressed as

$$\frac{d}{dt}\begin{bmatrix} \mathbf{I}^{\mathrm{mod}} \\ \mathbf{V}^{\mathrm{mod}} \end{bmatrix} = \mathbf{B}\begin{bmatrix} \mathbf{I}^{\mathrm{mod}} \\ \mathbf{V}^{\mathrm{mod}} \end{bmatrix} + \mathbf{C}\mathbf{v}_{\mathbf{S}} + \mathbf{D}\mathbf{v}_{\mathbf{L}}$$

A standard time integration routine for ODE's can be used to solve the above equations. In addition, we have included in our scheme, at any fixed



time step, an absolute cutoff of the wavelet coefficients with magnitude below a given threshold ε. This operation is justified by the theory of wavelet-based nonlinear approximations. As a result, the representation of the solution is sparse, and the time integration can be implemented by performing only the operations that involve nonvanishing coefficients. On one hand the time-stepping routine determines automatically the minimal set of coefficients needed to represent the solution for a given maximum level J and threshold ε. On the other hand, only these significant coefficients are used to compute the solution at the next time step. In conclusion, the numerical scheme allows to save computation time at no loss of accuracy.

We finish by showing a numerical example. Let us consider a scalar transmission line ($P = 1$) with no losses (i.e., $\mathbf{\Gamma R} = \mathbf{SG} = 0$) and with $\mathbf{\Gamma}(x) = 4^{-x}$, $\mathbf{S} = 1$. It is easy to show that in this case the signals propagate with a nonuniform speed $\nu(x) = 2^{-x}$. The figure depicts the location of the wavelet coefficients larger than a threshold $\varepsilon = 10^{-4}$ used to calculate the solution forced by a step voltage generator with rise time equal to $\tau = 0.3$. This waveform has two singularities in the first derivative. The figure shows that the significant wavelet coefficients track the location of the singularities as they propagate along the characteristic curves. The slope of these curves is exactely $1/\nu(x)$. This solution was generated by using biorthogonal B-spline wavelets and setting $j_0 = 5$, $J = 8$.

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